Let us note that the operation of a monoid

A new monoid homomorphism \( (x, y) \mapsto (x, y, z) \) for \( z \in G \) is defined as a mapping

Let us consider the following example:

An algebraic structure \( (G, \cdot, 1) \) is a monoid if it satisfies the following axioms:

- Closure: For all \( a, b \in G \), \( a \cdot b \in G \)
- Identity: There exists an element \( 1 \in G \) such that \( a \cdot 1 = 1 \cdot a = a \) for all \( a \in G \)
- Associativity: For all \( a, b, c \in G \), \( (a \cdot b) \cdot c = a \cdot (b \cdot c) \)

A monoid is called commutative if it satisfies the additional axiom:

- Commutativity: For all \( a, b \in G \), \( a \cdot b = b \cdot a \)

A monoid is called a group if it satisfies the additional axiom:

- Inverses: For every element \( a \in G \), there exists an element \( b \in G \) such that \( a \cdot b = b \cdot a = 1 \)

We refer to the following results:

- Theorem 1: Let \( (G, \cdot, 1) \) be a monoid. Then the set \( G \) endowed with the operation \( \cdot \) is a group if and only if for every \( a \in G \), there exists an element \( b \in G \) such that \( a \cdot b = 1 \).
- Theorem 2: Let \( (G, \cdot, 1) \) be a group. Then the set \( G \) endowed with the operation \( \cdot \) is a monoid.

Finally, we note that a monoid is a special case of a semigroup, which is a set equipped with an associative binary operation.

### References

